TOPIC 3
WAVES AND RADIATION

3.1 Waves

A **wave** may be defined as a disturbance that transfers energy from one place to another. Water waves, seismic waves, shock waves, sound and light are all examples of wave motion. All these except light require some material to travel through, while light involves electrical and magnetic disturbances that can travel most easily in a vacuum (empty space). Water waves, light and some seismic waves (e.g. S-waves) are **transverse waves** which means that the displacements are at right angles to the direction in which the waves are travelling. Sound waves, shock waves and some seismic waves (P-waves) are **longitudinal waves**: the disturbances are to and fro along the direction of wave travel, and involve variations in pressure and density (rather like stretching and squashing a spring).

The simplest sorts of waves involve a regular repeating displacement that can be represented as in Figure 3.1. Figure 3.1a shows how the displacement varies with position at one instant in time, while Figure 3.1b shows how the displacement varies with time at one position. The distance between two adjacent wave crests is the **wavelength**, usually represented by the Greek letter lambda, λ, and the time for one complete 'up and down' cycle is the **period**, T. The maximum displacement is the **amplitude**, A.

The number of complete cycles of the wave per unit time is the **frequency**, f. Frequency has SI units of s^{-1} (number of cycles per second) or hertz, Hz. 1 Hz = 1 s^{-1}. Frequency is the reciprocal of the period:

$$f = \frac{1}{T} \tag{3.1}$$

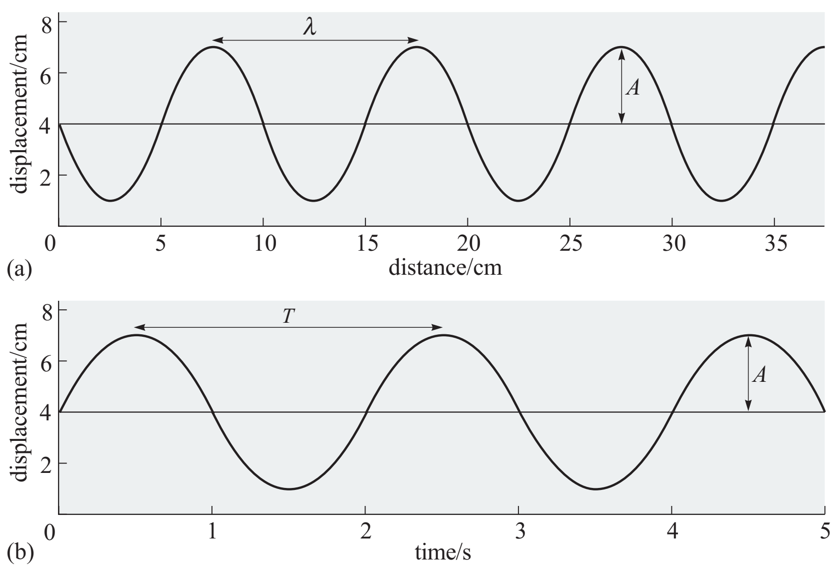

Figure 3.1 A simple wave represented as a graph of (a) displacement against distance and (b) displacement against time.

The wave speed v is the distance the wave travels divided by the time taken to travel. Comparing the two parts of Figure 3.1, the disturbance travels a distance λ in a time interval of T so

$$v = \frac{\lambda}{T}$$

The **wave equation** is more usually written

$$v = f\lambda \tag{3.2}$$

EXAMPLE 3.1

The musical note, middle C, has a frequency 256 Hz. What is the period of this note?

Sound waves travel in air at about 330 m s^{-1}. If you sing middle C, what is the wavelength of the wave you produce in the air?

Using Equation 3.1

$$T = \frac{1}{256\,\text{Hz}} = 3.9 \times 10^{-3}\ \text{s}$$

Rearranging Equation 3.2,

$$\lambda = \frac{v}{f} = \frac{330\,\text{m}\,\text{s}^{-1}}{256\,\text{Hz}} = 1.30\,\text{m}$$

EXAMPLE 3.2

BBC Radio 4 long-wave broadcasts on a wavelength 1500 m. The controls on some radio sets are labelled with frequency, and Radio 4 is found at 200 kHz (2.00×10^5 Hz). What is the speed of these radio waves in air?

From Equation 3.2, $v = 2.00 \times 10^5\ \text{Hz} \times 1500\ \text{m} = 3.00 \times 10^8\ \text{m}\,\text{s}^{-1}$.

QUESTION 3.1

Seismic P-waves travel through the Earth's crust at about $6.5 \times 10^3\ \text{m}\,\text{s}^{-1}$. If such a wave has a period of 2.0 s, what is its wavelength? What is the frequency of this wave?

3.2 Electromagnetic radiation

Nearly all the information we have about astronomical objects comes from studying the **electromagnetic radiation** we receive from them. This radiation includes light, radio waves and X-rays. When electromagnetic radiation is travelling through space it can be precisely described and understood in terms of *waves* (for example, the wavelength can be measured). All these waves involve electrical and magnetic disturbances that travel fastest in a vacuum, where they have a speed of $3.00 \times 10^8\ \text{m}\,\text{s}^{-1}$. This speed is denoted by c and is often referred to simply as the speed of light. The *wave equation* (Equation 3.2) for electromagnetic radiation is usually written

$$c = f\lambda \tag{3.3}$$

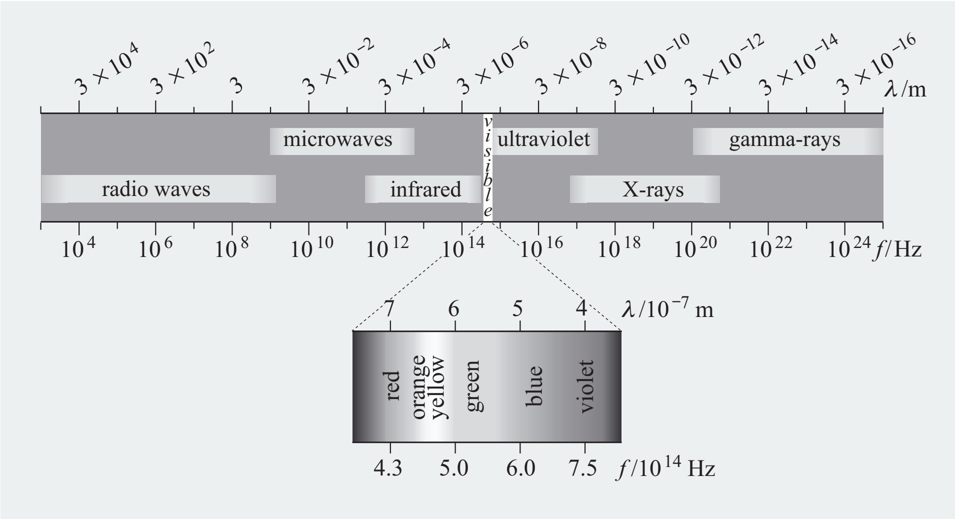

Figure 3.2 The electromagnetic spectrum.

As Figure 3.2 shows, electromagnetic waves have a vast range of frequencies and wavelengths. **Visible radiation** is that which we can see, and covers only a very narrow range of wavelengths. Radiation with wavelengths somewhat longer than those of red light is called **infrared**, and that with wavelengths shorter than violet light is **ultraviolet**. The longest-wavelength, lowest-frequency waves are the **radio** waves, and the radiations with the shortest wavelength and highest frequency are the **X-rays** and **gamma-radiation** (represented by the Greek gamma, γ). Collectively, the full range of waves is known as the **electromagnetic spectrum**. It is the different wavelengths and frequencies that determine the different properties of the waves, for example their ability to penetrate matter, or to cause heating or ionization, or the different ways in which instruments need to be designed to manipulate and detect them. As the range of values is so large, SI prefixes are generally used to express the very small and very large numbers required.

EXAMPLE 3.3

Red light emitted from hot interstellar hydrogen atoms has a wavelength 656 nm, where 1 nm = 1×10^{-9} m. What is the frequency of this radiation?

Wavelength $\lambda = 656 \times 10^{-9}\ \text{m} = 6.56 \times 10^{-7}\ \text{m}$ (see Topic 1). From Equation 3.3,

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m s}^{-1}}{6.56 \times 10^{-7}\ \text{m}} = 4.57 \times 10^{14}\ \text{Hz}$$

QUESTION 3.2

Cold interstellar hydrogen atoms emit radio waves with a frequency 1400 MHz, where 1 MHz = 1×10^{6} Hz. What is their wavelength?

3.3 Photons

When electromagnetic radiation interacts with matter, it can best be understood and described as a stream of particles. A particle of electromagnetic radiation is called a **photon**. Each photon has a distinct energy, and this determines how the photon behaves (for example, whether it can dislodge an electron from an atom to produce an ion).

The relationship between the wave and particle pictures of electromagnetic radiation is given by

$$E_{ph} = hf \tag{3.4}$$

where E_{ph} is the energy of the photon and h is the Planck constant 6.63×10^{-34} J s.

EXAMPLE 3.4

The red light from interstellar hydrogen atoms has a frequency 4.57×10^{14} Hz. What is the energy of one of its photons?

Using Equation 3.4,

$$\begin{aligned} E_{ph} &= 6.63 \times 10^{-34}\ \text{J s} \times 4.57 \times 10^{14}\ \text{Hz} \\ &= 3.03 \times 10^{-19}\ \text{J} \end{aligned}$$

As Example 3.4 shows, single photons have extremely small energy. Rather than expressing such energies in joules, we often use the non-SI unit of energy, the electronvolt (eV), where 1 eV = 1.60×10^{-19} J. However, a warning: for many calculations you need to express energy in SI units, so sometimes you may need to convert from eV to joules.

EXAMPLE 3.5

Express the photon energy of red hydrogen light (Example 3.4) in eV.

$$\begin{aligned} E_{ph} &= 3.03 \times 10^{-19}\ \text{J} \\ &= (3.03 \times 10^{-19} / 1.60 \times 10^{-19})\ \text{eV} \\ &= 1.89\ \text{eV} \end{aligned}$$

EXAMPLE 3.6

What is the frequency of electromagnetic radiation that has photons of energy 4.00 keV? Note that 1 keV = 10^3 eV.

As 1 eV = 1.60×10^{-19} J, then

$$\begin{aligned} E_{ph} &= 4.00 \times 10^3\ \text{eV} = (4.00 \times 10^3 \times 1.60 \times 10^{-19})\ \text{J} \\ &= 6.40 \times 10^{-16}\ \text{J} \end{aligned}$$

From Equation 3.4,

$$f = \frac{E_{ph}}{h} = \frac{6.40 \times 10^{-16}\ \text{J}}{6.63 \times 10^{-34}\ \text{J s}} = 9.65 \times 10^{17}\ \text{Hz}$$

QUESTION 3.3

Radio astronomers sometimes use telescopes with receivers tuned to a frequency of 2.7 GHz. Note that 1 GHz = 1×10^9 Hz. What is the energy of a single photon of the radiation they receive? Express your answer in joules and in eV.

QUESTION 3.4

A certain colour of violet light has photons with energy 3.20 eV. What is the frequency of this light?

3.4 Answers and comments for Topic 3

QUESTION 3.1

From Equations 3.1 and 3.2, $\lambda = v/f = vT = 6.5 \times 10^3\ \mathrm{m\,s^{-1}} \times 2.0\ \mathrm{s} = 1.3 \times 10^4$ m.

From Equation 3.1, $f = 1/T = 1/2.0\ \mathrm{s} = 0.50$ Hz.

QUESTION 3.2

Frequency $f = 1400 \times 10^6\ \mathrm{Hz} = 1.40 \times 10^9$ Hz

From Equation 3.3, $\lambda = c/f = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}/1.40 \times 10^9\ \mathrm{Hz} = 0.21$ m.

QUESTION 3.3

Using Equation 3.4,

$$E_{\mathrm{ph}} = hf = 6.63 \times 10^{-34}\ \mathrm{J\,s} \times 2.7 \times 10^9\ \mathrm{Hz}$$
$$= 1.79 \times 10^{-24}\ \mathrm{J}$$

To convert to units of eV,

$$E_{\mathrm{ph}} = (1.79 \times 10^{-24}/1.60 \times 10^{-19})\ \mathrm{eV}$$
$$= 1.12 \times 10^{-5}\ \mathrm{eV}$$

QUESTION 3.4

Following the method of Example 3.6,

$$E_{\mathrm{ph}} = 3.20\ \mathrm{eV} = 3.20 \times 1.60 \times 10^{-19)}\ \mathrm{J}$$
$$= 5.12 \times 10^{-19}\ \mathrm{J}$$

From Equation 3.4,

$$f = E_{\mathrm{ph}}/h = 5.12 \times 10^{-19}\ \mathrm{J}/6.63 \times 10^{-34}\ \mathrm{J\,s}$$
$$= 7.72 \times 10^{14}\ \mathrm{Hz}$$